SP222 The Charge-to-Mass Ratio of the Electron

Purpose   To observe the circular motion of an electron beam in a nearly-uniform magnetic field and deduce the charge-to-mass ratio of the electron from measurements of the orbital radius.

Reference   Tipler, Physics (4th edition), pp. 855-866 and 885-888.

Introduction   When a charged particle moves through a magnetic field, it experiences a magnetic force given by

$$\vec{F} = q\,\vec{v} \times \vec{B},$$

where q is the charge of the particle, $\vec{\boldsymbol{v}}$ its velocity, and $\vec{\boldsymbol{B}}$ the magnetic field. This force is always perpendicular to both the velocity and the magnetic field.

If the magnetic field is uniform and if the velocity of the particle is perpendicular to the field direction, the particle will move in a circular path at constant speed. By equating the magnitudes of the mass times the centripetal acceleration and the magnetic force, one easily shows that:

$$r = \frac{m}{q}\,\frac{v}{B};$$

thus, measuring the radius of the path for known values of v and B allows the charge-to-mass ratio q/m of the particle to be deduced.

Apparatus Notes   In this experiment, electrons are obtained from a hot filament, accelerated through a known potential difference, and caused to move in a circular path by the magnetic field produced by a pair of current-carrying coils. All of this takes place in a glass tube containing a low-density mercury vapor. As (a small fraction of) the electrons collide with the mercury atoms, they leave some of those atoms in excited states; when the atoms return to their ground states, they emit a blue light that makes the path of the electron beam visible. The radius of the path is set to a previously measured value by adjusting the accelerating voltage and the magnetic field until the beam strikes one of a number of reference "pegs" rigidly positioned in the tube. The speed of the electrons can be calculated from the accelerating voltage, and the magnetic field can be found by measuring the current through the coils. This information, together with the radius of the orbit, allows the charge-to-mass ratio e/m of the electron to be calculated.

Magnetic Field: The magnetic field is produced by two identical circular, current-carrying coils connected in series, placed with their axes coincident, and separated by a distance equal to one coil radius. Such an arrangement is known as a "Helmholtz" pair, and it produces a very uniform magnetic field between the coils. The field strength midway between the coils is given in terms of the geometry of the coils and the current flowing through them by:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R},$$

where μ_0 is the permeability of free space, N is the number of turns in each coil, I is the current through each coil, and R is the coil radius. For this apparatus, $N = 72$ and $R = 0.33$ m.

Electron Source: The electrons "boil" off a heated filament, and are accelerated toward a surrounding can or "plate" which is held at a positive potential with respect to the filament. If the electrons are emitted from the filament with zero kinetic energy, then they arrive at the plate with a kinetic energy equal to their charge times the potential difference between the filament and the plate:

$$K = \frac{1}{2} mv^2 = -e\,(V_{filament} - V_{plate}).$$

Some of the electrons escape from the can by passing through a small slit, as shown in Fig. 1. They then move in a circular path, and are made to strike one of the reference pegs by adjusting the strength of the magnetic field. The distances *d* from the filament to the outside edges of the five pegs are *0.0648 m*, *0.0775 m*, *0.0902 m*, *0.1030 m*, and *0.1154 m*. (Each of these distances represents the diameter of the electron orbit when the beam is set to strike the corresponding peg.) With the orbital radius given by the position of the peg, the speed given by the accelerating voltage, and the field given by the coil current, the value of *e/m* may be calculated.

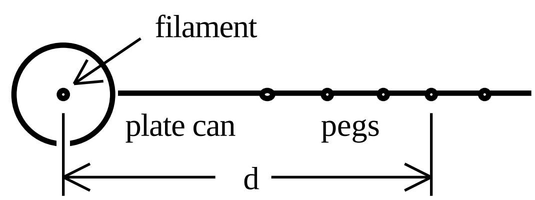

Fig. 1. Electron gun

The situation is slightly more complicated than this, for two reasons. First, the electrons have nonzero kinetic energy when they leave the filament, because of its high temperature. The average thermal kinetic energy of the emitted electrons is approximately 3/2 k_BT, where k_B is the Boltzmann constant and T is the filament temperature. For a typical temperature of 2000 K, this is only about 0.25 eV, however, which is small enough that it can be ignored. Second, the filament is not really an equipotential, because it is a resistive, current-carrying wire. As a result, there is a spread in the kinetic energy of the electrons arriving at the plate from different positions along the filament. This means a spread of velocities, which means a spread of orbital radii (the beam fans out slightly).

The impact of this spread is minimized by using a diode to half-wave-rectify the alternating current used to heat the filament as shown in Fig. 3. The whole filament is at ground potential during the zero-current half cycle, and (because of the choice of diode orientation) the filament potential increases relative to the plate potential during the other half-cycle. With this arrangement, the maximum kinetic energy of the electrons at the plate is $K = eV_{plate}$, and the maximum speed of electrons emerging from the slit in the plate is then $v = (2eV_{plate}/m)^{1/2}$. These "fastest" electrons follow the largest-radius path within the fanned-out beam, and if measurements are made with the farthest extent of the electron beam just hitting a peg on its outer side, then these electrons, with their known speed, are selected.

Earth's Magnetic Field: The Earth's magnetic field B_{Earth} is small, but it is not negligible in this experiment. Because the velocity of the electrons must be perpendicular to the (net) magnetic field they encounter, the field B_{coils} produced by the coils must be made either parallel or anti-parallel to the Earth's field. This is done by orienting the axis of the coils along the direction of the Earth's field—along a line from North to South and at an angle of about 22˚ with respect to the vertical. The current is passed through the coils in such a way as to cause their field to be directed upwards while the Earth's field is directed downwards. The net field is directed upwards and has a magnitude given by $B = B_{coils} - B_{Earth}$.

Calculation of e/m: Putting together the information contained in the last few paragraphs, we may write the charge-to-mass ratio of the electron in terms of the plate potential, the magnetic fields, and the orbital radius as follows:

$$\frac{e}{m} = \frac{2\,V_{plate}}{\left(B_{coils} - B_{Earth}\right)^2 r^2}.$$

<u>Procedure</u>

Part I. Setup

(1) The apparatus used in this experiment is expensive and rather delicate; it will have been set up and adjusted before you arrive. Do not change any wiring connections. Do not change the position of the Helmholtz coils or of the tube inside the coils. If some aspect of the setup seems wrong to you, consult the instructor.

(2) Verify that your computer is displaying the USNA Physics Laboratories screen. Click on the icon labeled Multi-Chan VAmeter. After about a minute, the Multi-Chan VAmeter startup screen will appear, where you will select the meters you wish to use. You will need one voltmeter and one ammeter for this experiment. Use the hand-shaped cursor to set the switches for Ch 1 and Ch 4 to ON, and the switches for the other channels to OFF. The voltmeter will be used to measure the plate potential V_{plate}, and the ammeter will be used to measure the coil current I. The value of the shunt resistor for the ammeter is printed on box containing the resistor. This resistance is approximately 20 mΩ. Find this value and enter it into the appropriate window on the Multi-Chan VAmeter startup screen. Then click on the button labeled Continue. The next screen will show your meters.

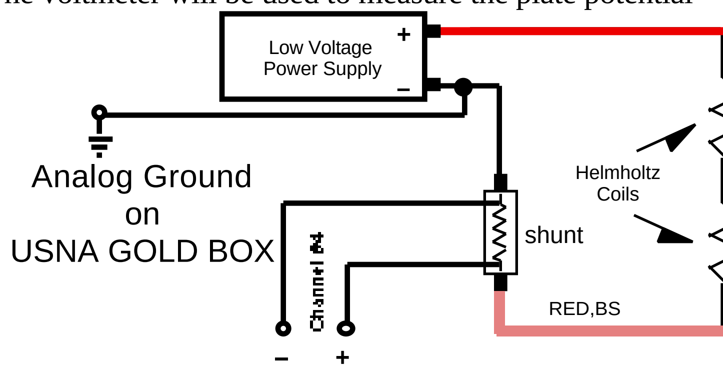

Figure 2 Helmholtz Coil Circuit

NOTE: You will use a plate potentials in the range 100 V. However, you must <u>never</u> apply a voltage greater than 15 V to the Gold Box inputs or expensive damage will occur. A voltage divider is used to reduce the potential to a safe level The voltage applied to the Gold Box imputs is 1/100 the plate voltage.. You must therefore multiply the recorded value by 100 to correct for this.

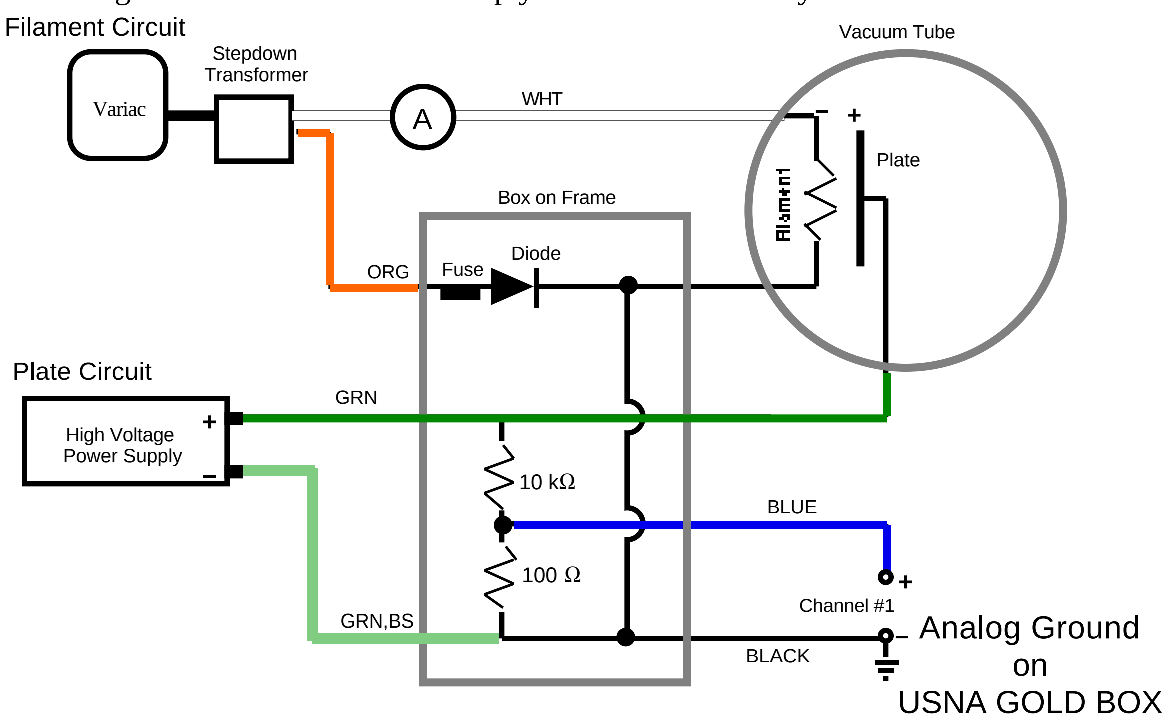

Figure 3 Filament and Plate Circuits

(3) Use the pull-down menu in the upper right-hand corner of the screen to return to Hypercard and then to open Data.Editor. Then you can use the pull-down Window menu to switch back and forth between Multi-Chan VAmeter and Data.Editor whenever you want.

Part II. Filament current adjustment

(1) Using the Pasco Model SF-9585 High Voltage Supply, set the plate potential to about 100 V. Turn the variac down to 0% and then turn its power switch on.

(2) Slowly increase the filament voltage, by turning up the variac, until the plate current reaches 7 mA or until a faint blue beam is visible in the tube. Work with the lowest filament current possible, to extend the tube life. Never exceed a filament current of 4.5 amperes, read on the A.C. ammeter.

NOTE: The plate current can only be read while the Meter Select button is depressed on the Model SF-9585 High Voltage Supply and the yellow LED beside the "mA" label is lighted.

Part III. Field current adjustment

(1) Increase the current through the Helmholtz coils by adjusting the controls on the Pasco Model SF-9584 Low Voltage AC/DC Power Supply. As you do, the electron beam should curve in a tighter circle as the current is increased. The maximum current you should use is 7 to 8 amperes.

(2) For best operation, set the DC Voltage Adjust control on the power supply to its upper limit, and use the DC Current Adjust control to adjust the coil current.

(3) Turn the coil current down to zero between measurements. Otherwise, the coils will heat up and their resistances will change. This will cause a significant error in the measured current, because the value of the shunt resistance entered into the software will be incorrect.

Part IV. Measurements

(1) Set the plate potential to about 100 V. Adjust the coil current until the electron beam strikes the outer edge of the farthest peg. Use the hand-shaped cursor to click on the button labeled Store Data to measure and record the plate potential and the coil current simultaneously.

(2) Increase the coil current until the electron beam strikes the outer edge of the next peg. Click on Store Data. Repeat for each of the remaining pegs. When you have finished, save the data for this plate potential by clicking on the button labeled Save Data.

Part V. Analysis

(1) Manipulate the equations displayed earlier to show that the current I through the coils can be related to the plate potential V_{plate}, the coil radius R and turns number N, the Earth's field B_{Earth}, the orbital radius r, and the charge-to-mass ratio e/m by:

$$I = \frac{R}{8\,\mu_0\,N}\sqrt{\frac{250\,V_{plate}\,m}{e}}\,\frac{1}{r} + \frac{R}{8\,\mu_0\,N}\sqrt{125}\,B_{Earth}\,.$$

(2) For each accelerating potential, plot I *vs* $1/r$ and fit a straight line to the data. Note that you will need to add the radii to your previously saved data files. You can do this using either Excel the Keyboard Entry feature within Data.Editor. From the slope and intercept of this line, deduce e/m and B_{Earth}, respectively. From the uncertainties in the slope and intercept, estimate the uncertainties in e/m and B_{Earth}, respectively. Compare your results to the accepted values.